An Efficient and Reasonably Simple Solution to the Perspective-Three-Point Problem

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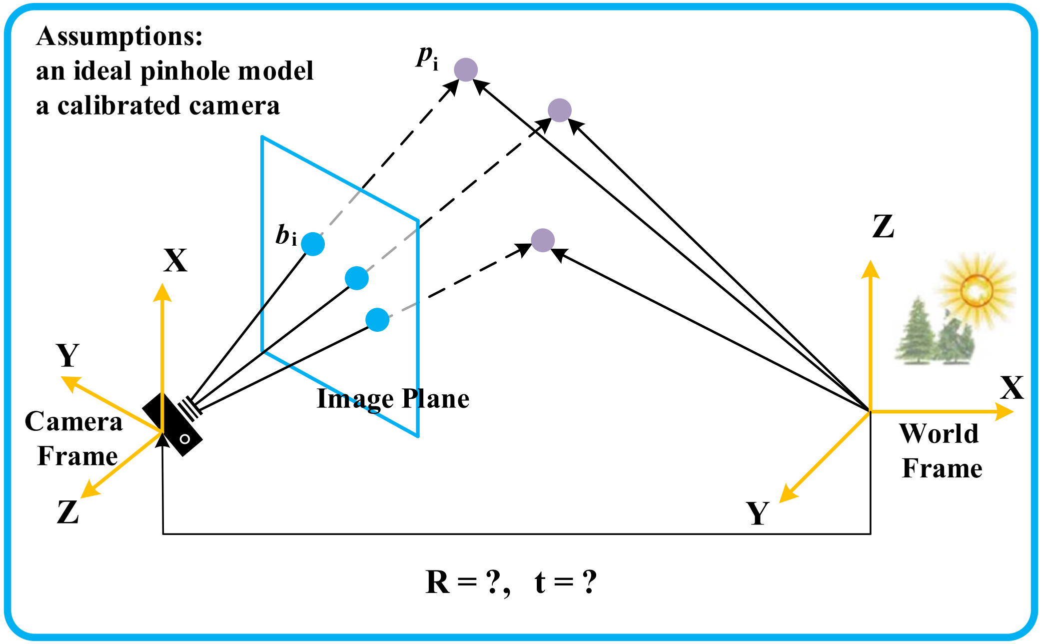
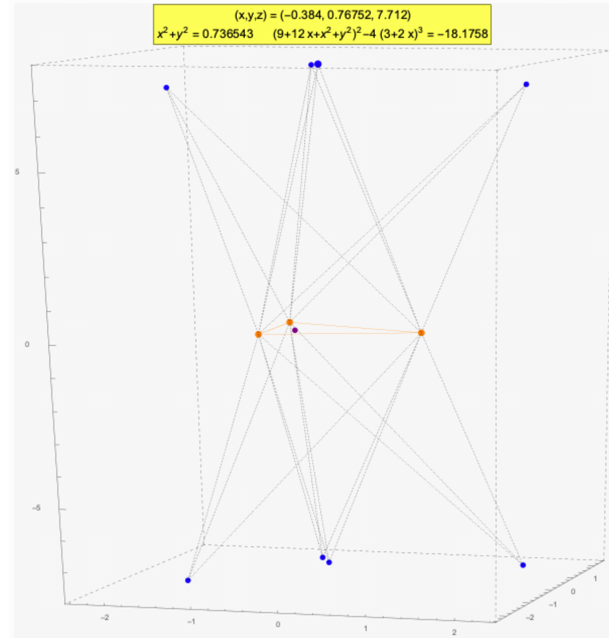
An efficient and simple solution to P3P





Problem Formulation: Given the corresponding relationship between the 3D model and its projections on the image, our goal is to solve for the pose of the camera w.r.t the world frame

The core: bi and pi are known,

R and t = ?

Classification of this problem:

Three points, the P3P problem,

N points, the PnP problem.

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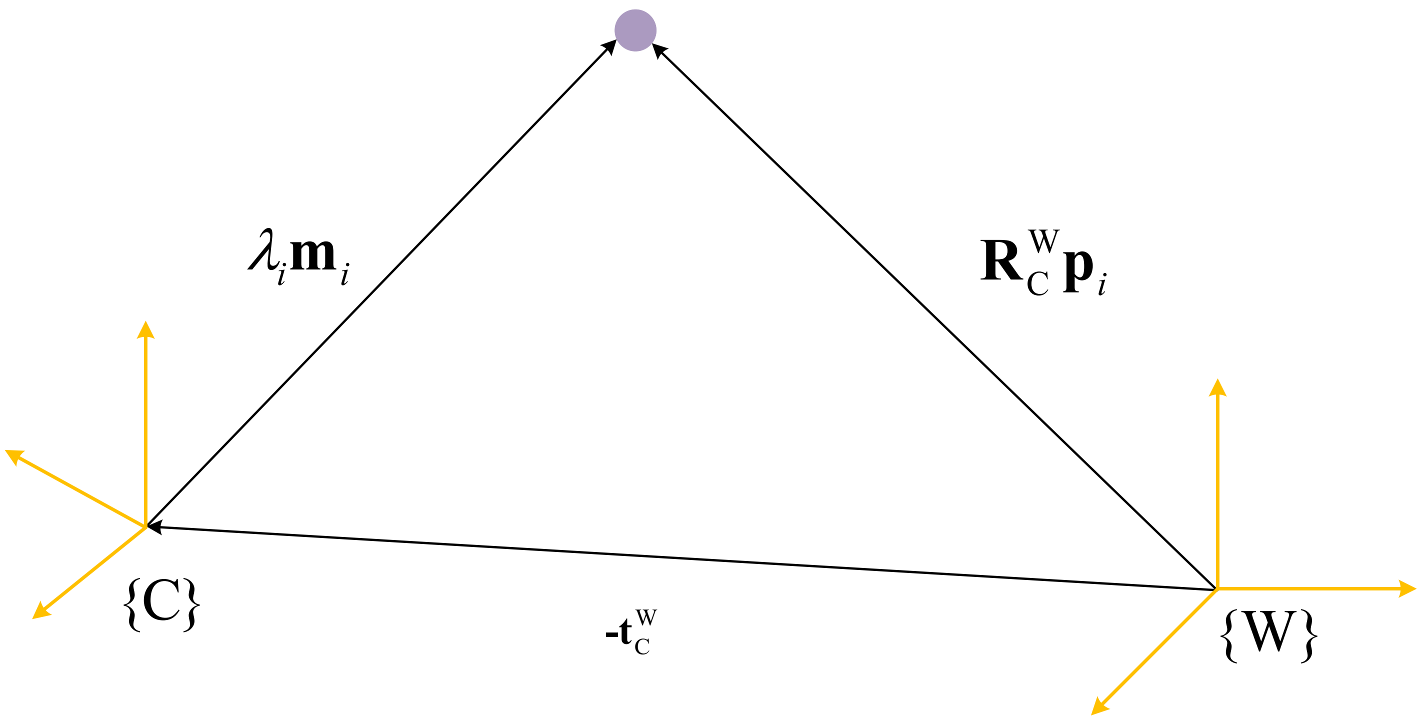

Model: $-\mathbf{t}_C^W + \lambda_i \mathbf{m}_i - \mathbf{R}_C^W \mathbf{p}_i = \mathbf{0}, \quad i \in \{1,2,3\}$

$$\lambda_i \mathbf{m}_i = [\mathbf{R}_C^W \mid \mathbf{t}_C^W] \begin{bmatrix} \mathbf{p}_i \\ 1 \end{bmatrix}, \quad i \in \{1,2,3\}$$

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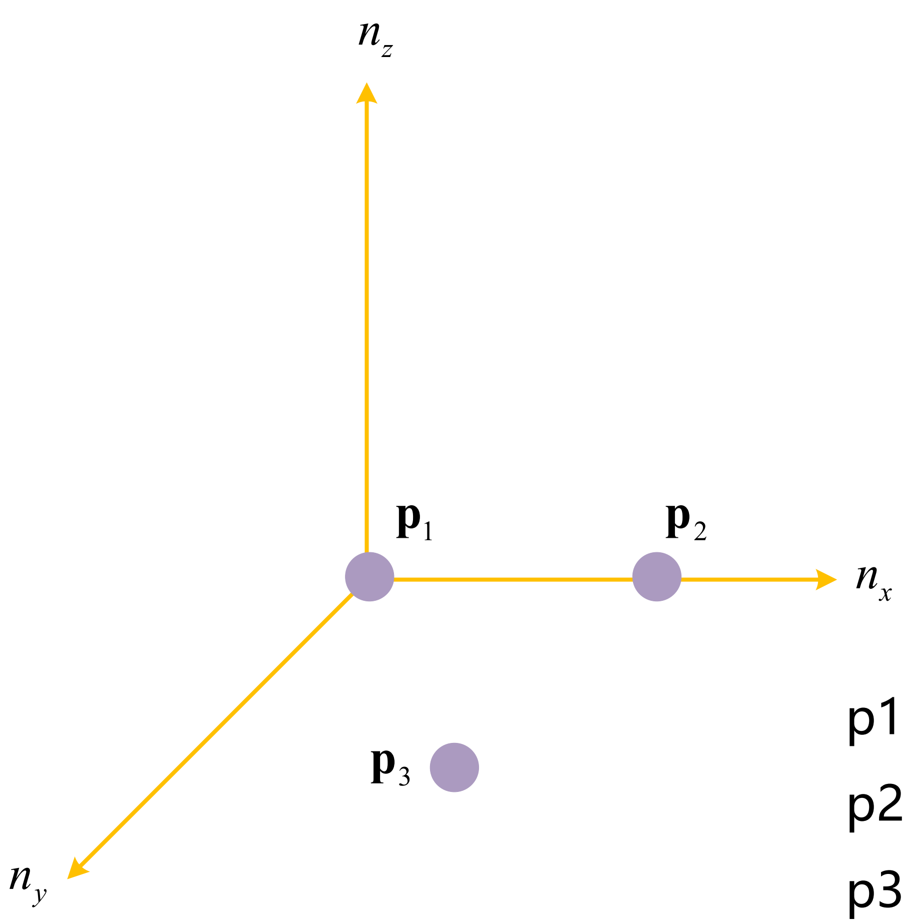
1、Definition of a New Intermediate Coordinate Frame



$$\mathbf{n}_x = \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|},$$

$$\mathbf{n}_z = \frac{\mathbf{n}_x \times (\mathbf{p}_3 - \mathbf{p}_1)}{\left\| \mathbf{n}_x \times (\mathbf{p}_3 - \mathbf{p}_1) \right\|},$$

$$\mathbf{n}_y = \mathbf{n}_z \times \mathbf{n}_x .$$

p1=[X1, Y1, Z1];		p1=[0, 0, 0];
p2=[X2, Y2, Z2];	→	p2=[a, 0, 0];
p3=[X3, Y3, Z3].		p3=[b, c, 0].

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2、A Novel Parameterization Method

$$\lambda_i \mathbf{m}_i = [\mathbf{R}_C^O \mid \mathbf{t}_C^O] \begin{bmatrix} \mathbf{p}_i \\ 1 \end{bmatrix}, \quad i \in \{1,2,3\}$$

$$\mathbf{m}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \mathbf{R}_C^O = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}, \mathbf{t}_C^O = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Matrix-vector form

$$\mathbf{M}_1 \mathbf{v}_1 = \mathbf{0}, \quad \mathbf{M}_1 \in \mathbb{R}^{9\times 12},$$

$$\mathbf{v}_1 = [r_1, r_2, r_4, r_5, r_7, r_8, t_x, t_y, t_z, \lambda_1, \lambda_2, \lambda_3]^T$$

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2、A Novel Parameterization Method

$$
\mathbf{M}_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -x_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -y_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -z_1 & 0 & 0 \\
a & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -x_2 & 0 \\
0 & 0 & a & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -y_2 & 0 \\
0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 1 & 0 & -z_2 & 0 \\
b & c & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -x_3 \\
0 & 0 & b & c & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -y_3 \\
0 & 0 & 0 & 0 & b & c & 0 & 0 & 1 & 0 & 0 & -z_3
\end{bmatrix}
$$

基于点特征的位姿估计方法

2、A Novel Parameterization Method

$$\lambda_i \mathbf{m}_i = [\mathbf{R}_C^O \mid \mathbf{t}_C^O] \begin{bmatrix} \mathbf{p}_i \\ 1 \end{bmatrix}, \quad i \in \{1,2,3\}$$

$$\lambda_3 \begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \begin{bmatrix} b \\ c \\ 0 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}.$$

$$x_3 = \frac{r_1 b + r_2 c + t_x}{r_7 b + r_8 c + t_z},$$

$$y_3 = \frac{r_4 b + r_5 c + t_y}{r_7 b + r_8 c + t_z}.$$

Similarly

$$\frac{x_1}{z_1} = \frac{t_x}{t_z}, \quad \frac{y_1}{z_1} = \frac{t_y}{t_z},$$

$$\frac{x_2}{z_2} = \frac{a r_1 + t_x}{a r_7 + t_z}, \quad \frac{y_2}{z_2} = \frac{a r_4 + t_y}{a r_7 + t_z}.$$

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2、A Novel Parameterization Method

$$\mathbf{M}_2\mathbf{v}_2 = \mathbf{0}, \quad \mathbf{M}_2 \in \mathbb{R}^{6\times 9}$$

$$\mathbf{M}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{x_1}{z_1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{y_1}{z_1} \\ a & 0 & 0 & 0 & -a\frac{x_2}{z_2} & 0 & 1 & 0 & -\frac{x_2}{z_2} \\ 0 & 0 & a & 0 & -a\frac{y_2}{z_2} & 0 & 0 & 1 & -\frac{y_2}{z_2} \\ b & c & 0 & 0 & -b\frac{x_3}{z_3} & -c\frac{x_3}{z_3} & 1 & 0 & -\frac{x_3}{z_3} \\ 0 & 0 & b & c & -b\frac{y_3}{z_3} & -c\frac{y_3}{z_3} & 0 & 1 & -\frac{y_3}{z_3} \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} r_1 \\ r_2 \\ r_4 \\ r_5 \\ r_7 \\ r_8 \\ t_x \\ t_y \\ t_z \end{bmatrix}$$

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2、A Novel Parameterization Method

$$\mathbf{M}_2\mathbf{v}_2 = \mathbf{0}, \quad \mathbf{M}_2 \in \mathbb{R}^{6\times 9}$$

15 parameters to 3 parameters

$$\mathbf{v}_2 = \alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2 + \alpha_3\mathbf{u}_3$$

We can use $\frac{1}{\alpha_3}$ to further reduce one parameter.

The elements of $\mathbf{R}_C^O, \mathbf{t}_C^O$ can be zero.

Return to $\lambda_i\mathbf{m}_i = [\mathbf{R}_C^O \mid \mathbf{t}_C^O]\begin{bmatrix}\mathbf{p}_i \\ 1\end{bmatrix}, \quad i \in \{1,2,3\}$

Multiply $\frac{1}{\lambda_1}$ at both sides:

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 \\ \lambda_1 y_1 \\ \lambda_1 z_1 \end{bmatrix}$$

2、A Novel Parameterization Method

$$\frac{x_2}{z_2}=\frac{a\frac{r_1}{d_1}+x_1}{a\frac{r_7}{d_1}+z_1},\quad \frac{y_2}{z_2}=\frac{a\frac{r_4}{d_1}+y_1}{a\frac{r_7}{d_1}+z_1},$$

$$\frac{x_3}{z_3}=\frac{b\frac{r_1}{d_1}+c\frac{r_2}{d_1}+x_1}{b\frac{r_7}{d_1}+c\frac{r_8}{d_1}+z_1},\quad \frac{y_3}{z_3}=\frac{b\frac{r_4}{d_1}+c\frac{r_5}{d_1}+y_1}{b\frac{r_7}{d_1}+c\frac{r_8}{d_1}+z_1}.$$

$$\mathbf{M}_3\mathbf{v}_3=\mathbf{d},\quad \mathbf{M}_3\in\mathbb{R}^{4\times 6},\mathbf{d}\in\mathbb{R}^4$$

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2、A Novel Parameterization Method

$$\mathbf{M}_3\mathbf{v}_3 = \mathbf{d}, \quad \mathbf{M}_3 \in \mathbb{R}^{4\times 6}, \mathbf{d} \in \mathbb{R}^4$$

$$\mathbf{M}_3 = \begin{bmatrix} a & 0 & 0 & 0 & -a\frac{x_2}{z_2} & 0 \\ 0 & 0 & a & 0 & -a\frac{y_2}{z_2} & 0 \\ b & c & 0 & 0 & -b\frac{x_3}{z_3} & -c\frac{x_3}{z_3} \\ 0 & 0 & b & c & -b\frac{y_3}{z_3} & -c\frac{y_3}{z_3} \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} \frac{x_2 z_1}{z_2} - x_1 \\ \frac{y_2 z_1}{z_2} - y_1 \\ \frac{x_3 z_1}{z_3} - x_1 \\ \frac{y_3 z_1}{z_3} - y_1 \end{bmatrix}$$

$$\mathbf{v}_3 = [s_1, s_2, s_3, s_4, s_5, s_6]^T$$

$$s_1 = \frac{r_1}{d_1}, s_2 = \frac{r_2}{d_1}, s_3 = \frac{r_4}{d_1},$$

$$s_4 = \frac{r_5}{d_1}, s_5 = \frac{r_7}{d_1}, s_6 = \frac{r_8}{d_1},$$

↓

$$\mathbf{v}_3 = \beta_1\mathbf{b}_1 + \beta_2\mathbf{b}_2 + \mathbf{b}_3$$

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2、A Novel Parameterization Method

$$\mathbf{M}_4 = [\mathbf{M}_3, -\mathbf{d}] \qquad \mathbf{M}_4 = [\mathbf{M}_{4l} \mid \mathbf{M}_{4r}]$$

$$\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\mathbf{M}_{4l}^{-1}\mathbf{M}_{4r} \\ \mathbf{I}_{3\times3} \end{bmatrix}$$

$$[\mathbf{M}_3, -\mathbf{d}][\begin{matrix} \mathbf{v}_3 \\ 1 \end{matrix}] = \mathbf{M}_4 \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ 1 \end{bmatrix}$$

$$= [\mathbf{M}_{4l} \mid \mathbf{M}_{4r}] \begin{bmatrix} -\mathbf{M}_{4l}^{-1}\mathbf{M}_{4r} \\ \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ 1 \end{bmatrix}$$

$$= (-\mathbf{M}_{4r} + \mathbf{M}_{4r}) \begin{bmatrix} \beta_1 \\ \beta_2 \\ 1 \end{bmatrix} = \mathbf{0}_{4\times1},$$

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2、A Novel Parameterization Method

$$\mathbf{b}_1 = \begin{bmatrix} b_{1,1} \\ b_{2,1} \\ b_{3,1} \\ b_{4,1} \\ b_{5,1} \\ b_{6,1} \end{bmatrix} = \begin{bmatrix} \frac{x_2}{z_2} \\ \frac{b(\frac{x_3}{z_3} - \frac{x_2}{z_2})}{c} \\ \frac{y_2}{z_2} \\ \frac{b(\frac{y_3}{z_3} - \frac{y_2}{z_2})}{c} \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} b_{1,2} \\ b_{2,2} \\ b_{3,2} \\ b_{4,2} \\ b_{5,2} \\ b_{6,2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{x_3}{z_3} \\ 0 \\ \frac{y_3}{z_3} \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} b_{1,3} \\ b_{2,3} \\ b_{3,3} \\ b_{4,3} \\ b_{5,3} \\ b_{6,3} \end{bmatrix} = \begin{bmatrix} \frac{-\frac{x_1}{z_1} + \frac{x_2}{z_2}}{a / z1} \\ \frac{a(-\frac{x_1}{z_1} + \frac{x_3}{z_3}) + b(\frac{x_1}{z_1} - \frac{x_2}{z_2})}{ac / z1} \\ \frac{-\frac{y_1}{z_1} + \frac{y_2}{z_2}}{a / z1} \\ \frac{a(-\frac{y_1}{z_1} + \frac{y_3}{z_3}) + b(\frac{y_1}{z_1} - \frac{y_2}{z_2})}{ac / z1} \\ 0 \\ 0 \end{bmatrix}.$$

$\beta_1 = s_5$

$\beta_2 = s_6$

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3、Solving for the pose

$$\mathbf{R}^T\mathbf{R} = \mathbf{I}_{3\times 3}$$
$$\det(\mathbf{R}) = 1$$

$$\begin{aligned} r_1 r_2 + r_4 r_5 + r_7 r_8 &= 0, \\ r_1^2 + r_4^2 + r_7^2 - r_2^2 - r_5^2 - r_8^2 &= 0, \\ r_1^2 + r_4^2 + r_7^2 = r_2^2 + r_5^2 + r_8^2 &= 1. \end{aligned}$$

$$\lambda_1 = \sqrt{\frac{2}{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2 + s_6^2}}$$

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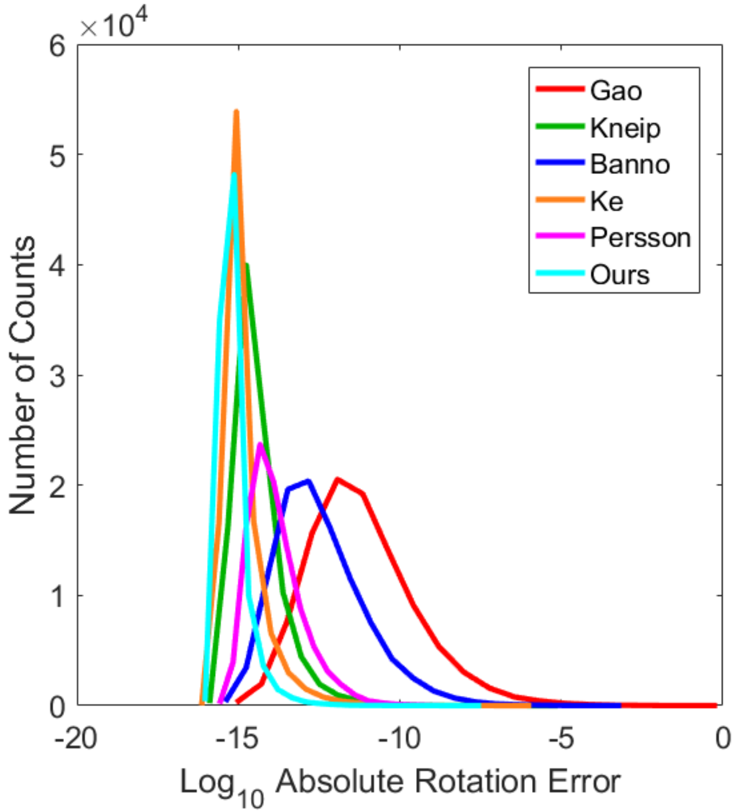
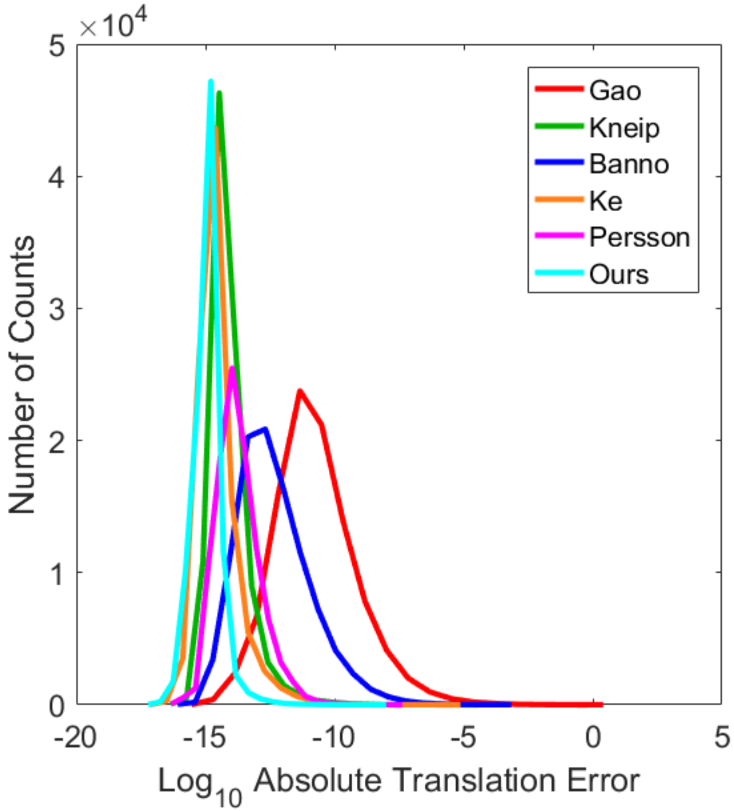
4、Results





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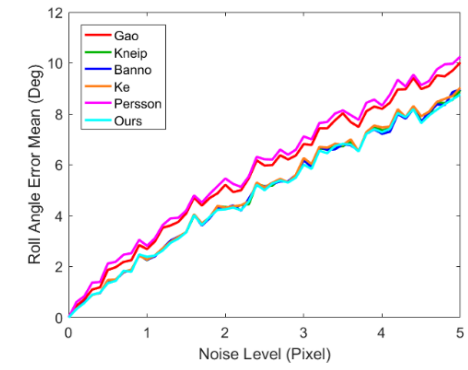
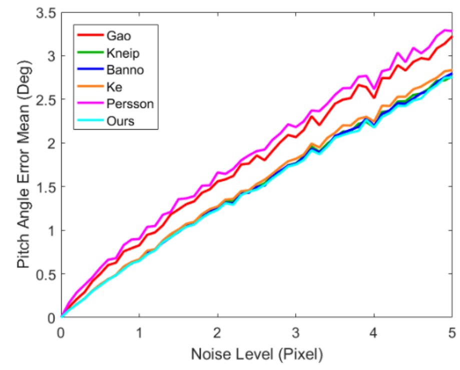
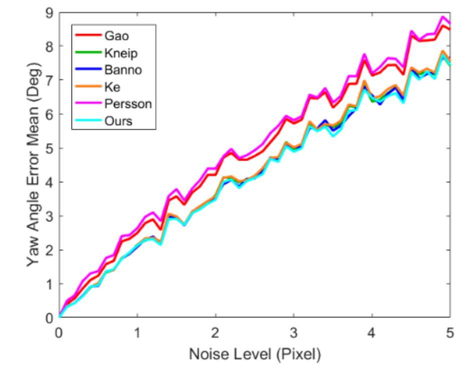
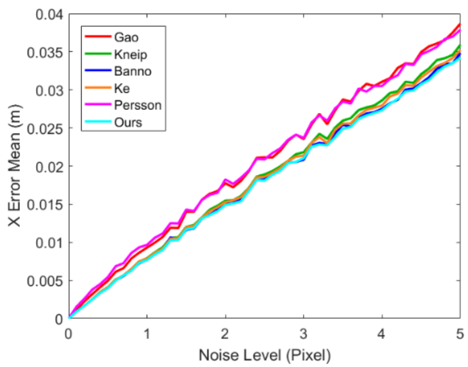
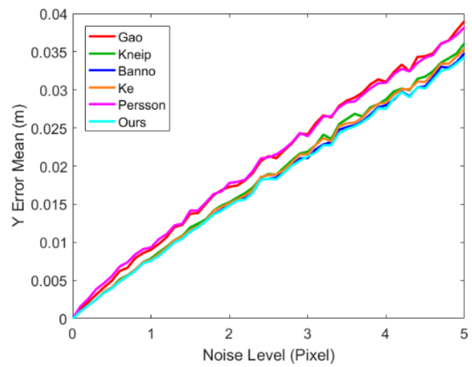
4、Results

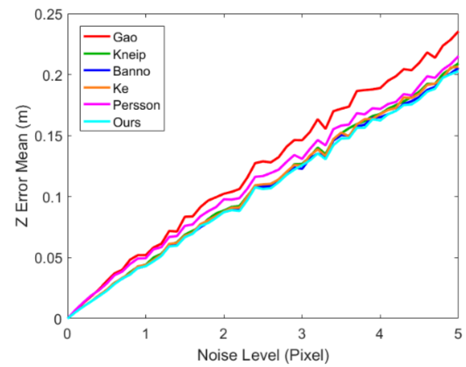












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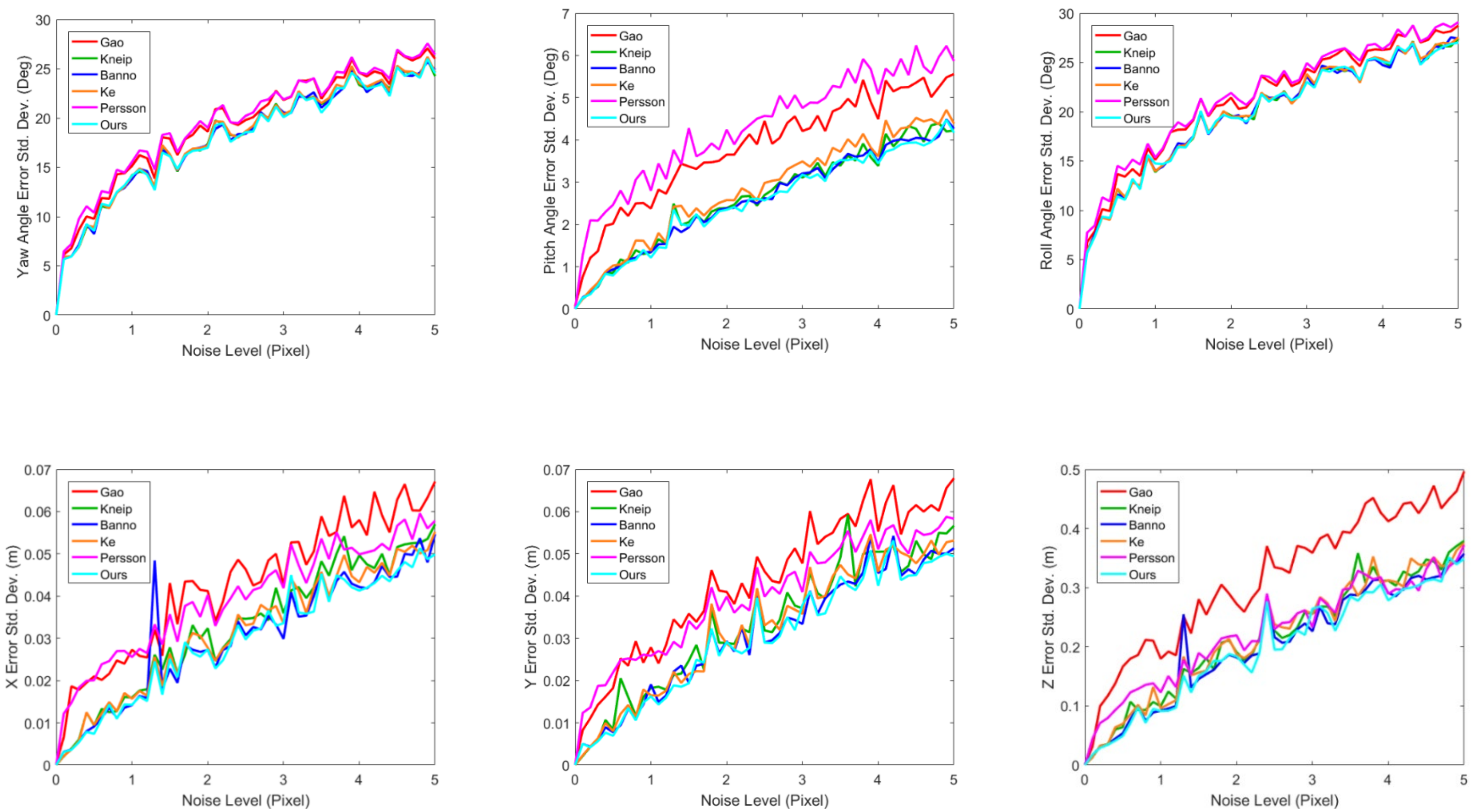

Thanks for Your Attention